Electromagnetic wave propagation in quasi-one-dimensional comb-like structures made up of dissipative negative-phase-velocity materials

2006 J. Phys.: Condens. Matter 18 3683

(http://iopscience.iop.org/0953-8984/18/15/014)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **18** (2006) 3683–3690 doi:10.1088/0953-8984/18/15/014

Electromagnetic wave propagation in quasi-one-dimensional comb-like structures made up of dissipative negative-phase-velocity materials

Gregorio H Cocoletzi[1,2], **L Dobrzynski**[1], **B Djafari-Rouhani**[1], **H Al-Wahsh**[1,3] **and D Bria**[1,4]

[1] Institut d'Electronique, de Microelectronique et de Nanotechnologie (IEMN), UMR CNRS 8520, Université des Sciences et Technologies de Lille, 59655 Villeneuve d'Ascq Cédex, France
[2] Instituto de Física, Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla, 72570 Puebla, Mexico
[3] Faculty of Engineering, Benha University, 11241 Cairo, Egypt
[4] Faculté des Sciences, Université Mohamed I, 60000 Oujda, Morocco

Received 26 January 2006
Published 30 March 2006
Online at stacks.iop.org/JPhysCM/18/3683

Abstract
We study the propagation of electromagnetic waves in infinite lines with grafted finite lines in a frequency regime, where the grafted lines have negative phase velocity. The physical properties of such a structure with negative phase velocity materials have not been investigated before, to the best of our knowledge. The theory uses a Green's function formalism. For a single resonator structure we calculate the transmission intensity, and for periodic systems, we determine the transmission intensities and the miniband structures as functions of the number of resonators. In a single resonator structure, resonances are manifested as dips in the transmission, which evolve into stopping minigaps for periodic systems. The dispersion relations of collective normal modes exhibit new allowed minibands or stopping minigaps for the electromagnetic wave propagation, which depend on the number of resonators and the resonator size. These results are in agreement with the features of the transmission spectra. We also investigate the effects of dissipation on the transmission spectra of such structures.

1. Introduction

Negative phase velocity materials with negative permittivity (ε) and negative permeability (μ), which were predicted many years ago in the pioneering work of Veselago [1], have attracted a growing interest during the last few years [2–7]. They are theoretically predicted to offer many new potential applications [8, 9] in optical and electromagnetic devices, thanks to their unusual properties. Assuming the possibility of realizing such materials under the form of layered media, recent studies [10–14] have investigated the photonic band structure of one-dimensional

(1D) layered structures composed of alternating layers of positive and negative phase velocity materials. Negative phase velocity materials have been used to realize an absolute photonic gap [14] (and also to widen this stop band) with 1D photonic crystals independently of the incidence angles and the polarization of the optical wave.

The propagation of electromagnetic waves in 1D systems such as superlattices [15] and quasi-1D comb-like structures [16, 17] is another issue of interest in electromagnetic devices. In these composite systems, the contrast in dielectric (ε) properties between the constituent materials is emerging as a critical parameter in determining the existence of electromagnetic gaps. Let us recall that the comb-like structure is composed of back-bone (substrate) waveguiding along which finite side branches are grafted periodically. The materials forming such a structure were supposed until now not to be magnetic ($\mu = 1$) and to have a positive and frequency independent ε. In our present paper, we investigate the novel electromagnetic properties of the comb-like structure when one of the material constituting this quasi-1D structure has simultaneously negative frequency dependent ε and μ in a certain range of frequency. Our studies differ from those of [16] as we concentrate on a small frequency region where the resonators have negative phase velocity. We investigate the new gaps which do not exist for positive phase velocity resonators. We address also for the first time the effects of dissipation on the electromagnetic properties of such materials. The structures with dissipative negative-phase velocity materials proposed here should be of interest for future investigations and applications.

The transmission coefficient and the dispersion relation are calculated in the framework of a Green's function method [17]. A brief presentation of the model and of the method of calculation is presented in section 2. Section 3 contains the numerical illustrations as well as the discussion of the transmission coefficients for comb-like structures with dissipative negative phase velocity materials. Conclusions are given in section 4.

2. Theory

Let us consider a quasi-1D composite system made up of a finite segment of length d_2 grafted on an infinite waveguide line (see figure 1). The infinite waveguide is along the z-direction. In this communication we will address the problem of determining the transmission of electromagnetic waves and the miniband structure. To calculate the optical properties we apply the interface response theory. We first construct the Green's function of an infinite 1D medium parallel to the z-axis. We consider the lateral dimensions of such a 1D medium to be small compared to the wavelength of the electromagnetic waves. In this limit the wave equation that satisfies the Green's function $G_i(z - z')$ is

$$\frac{F_i}{\alpha_i \mu_i}\left[\frac{\partial^2}{\partial z^2} + \alpha_i^2\right] G_i(z, z') = \delta(z - z'), \tag{1}$$

where $F_i = (\omega^2/c^2)\varepsilon_i\mu_i$, $\alpha_i = (\omega/c)\sqrt{\varepsilon_i\mu_i}$, ω is the angular frequency of the electromagnetic radiation, $\varepsilon_i(\omega)$ and $\mu_i(\omega)$ are the dielectric function and magnetic permeability of medium i, respectively, for homogeneous isotropic media. The value of α_i is valid for both transverse electric (TE) and transverse magnetic (TM) polarizations. In what follows we will consider only electromagnetic modes which satisfy the $H = 0$ [16, 17] boundary condition.

The Green's function which solves equation (1) has the form [16, 17]

$$G_i(z - z') = \frac{\mathrm{e}^{\mathrm{i}\alpha_i|z-z'|}}{2\mathrm{i}F_i}. \tag{2}$$

As stated above, we use the interface response theory to investigate optical properties of comb structures. We first consider a quasi-1D composite system formed out of a finite segment

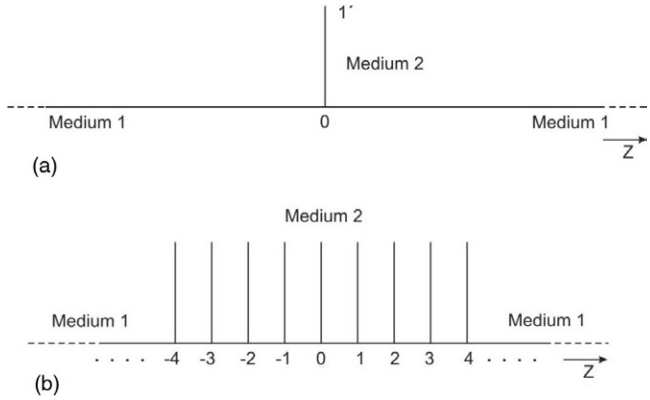

Figure 1. (a) Waveguide with a single grafted segment of length d_2. (b) Waveguide with a periodic array of grafted segments of length d_2 distant from each other by a length d_1. Each medium i is characterized by the dielectric function ε_i and magnetic permittivity μ_i.

of length d_2 grafted on an infinite waveguide line. Within the interface response theory, the corresponding interface Green's function of the system made up of two semi-infinite lines of the same dielectric material 1 and a segment of dielectric material 2 of finite length d is obtained by constructing first the inverse surface Green's function which takes the form [16, 17]

$$g^{-1}(0,0) = 2\mathrm{i}\alpha_i + \alpha_2\tan(\alpha_2 d_2). \tag{3}$$

To calculate the transmission coefficient we take the formula of Vasseur *et al* [16],

$$T = \frac{1}{1 + (\varepsilon_2\mu_1/4\varepsilon_1\mu_2)\tan^2(\alpha_2 d_2)}. \tag{4}$$

In this formula we note that for frequency independent dielectric and magnetic responses, T vanishes when the resonance condition

$$\alpha_2 d_2 = \frac{2m+1}{2}\pi, \tag{5}$$

is met, with m being a positive integer.

Let us consider an infinite comb-like structure composed of finite segments (medium 2) of length d_2 grafted periodically with spacing d_1 on an infinite substrate (medium 1). The infinite line can be modelled as an infinite number of segments of length d_1 in the z-direction, each one being pasted to two neighbours. The inverse surface Green's function of the composite is an infinite banded matrix $[g_\infty(M,M)]^{-1}$ defined in the interface domain constituted of n sites which are the connection points between finite segments. Taking into account the translational symmetry we write [16, 17]

$$\left[g_\infty(k,M,M)\right]^{-1} = \frac{2\alpha_1}{\sin(\alpha_1 d_1)}[-\xi + \cos(kd_1)] \tag{6}$$

for the Fourier transform of the inverse of the Green's function. In this equation, k is the 1D propagation wavevector, which is real for passing bands and imaginary for stopping gaps. In equation (6)

$$\xi = \cos(\alpha_1 d_1) - \frac{1}{2}\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}}\sin(\alpha_1 d_1)\tan(\alpha_2 d_2). \tag{7}$$

The dispersion relation of the collective normal modes in the infinite periodic comb-like waveguide is obtained from equations (6) and (7), and is expressed in the form

$$\cos(kd_1) = \xi. \tag{8}$$

Combining equations (7) and (8), we obtain

$$\cos(kd_1) = \cos(\alpha_1 d_1) - \frac{1}{2}\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}}\sin(\alpha_1 d_1)\tan(\alpha_2 d_2). \tag{9}$$

In the absence of dissipation, the dispersion relation (9) can be solved in the following way. The right-hand side of equation (9) is evaluated for any value of ω. If its absolute value is smaller than 1, one can obtain a real solution for k, i.e., the corresponding wave propagates along the axis of the comb-like structure and ω belongs to a pass-band. Otherwise, k becomes a complex number, the wave cannot propagate and ω belongs to gaps of the comb-like structure.

Let us then consider an incident electromagnetic wave [2, 5] coming from $z = -\infty$,

$$u(z) = \mathrm{e}^{\mathrm{i}\alpha_1 z}. \tag{10}$$

Following the expression detailed in [16, 17], the corresponding transmission coefficient is

$$T = \left|\frac{2\sin(\alpha_1 d_1)\left(t^2-1\right)t^N}{\left(1-t\mathrm{e}^{\mathrm{i}\alpha_1 d_1}\right)^2 - t^{2N}\left(t-\mathrm{e}^{\mathrm{i}\alpha_1 d_1}\right)^2}\right|^2 \tag{11}$$

where N is the number of resonators and $t = \mathrm{e}^{\mathrm{i}kd_1}$, or equivalently

$$t = \begin{cases} \xi - \sqrt{\xi^2-1}, & \xi < 1 \\ \xi + \sqrt{\xi^2-1}, & \xi < -1 \\ \xi + \mathrm{i}\sqrt{1-\xi^2}, & -1 < \xi < 1. \end{cases} \tag{12}$$

3. Results

In this section, we shall give some specific illustrations of our theoretical results. We shall study the effect of the introduction of negative velocity resonators on the frequency spectrum and the transmission coefficients. In our calculations, we illustrate the band structure and the transmission coefficient for a comb-like structure in which the dielectric, $\varepsilon_i(\omega)$, and magnetic, $\mu_i(\omega)$, responses are frequency dependent, and take the following forms:

$$\varepsilon_i(\omega) = 1 - \frac{\omega_{pi}^2}{\omega(\omega+\mathrm{i}\nu)}, \qquad \mu_i(\omega) = 1 - \frac{f\omega^2}{\omega(\omega+\mathrm{i}\nu)-\omega_i^2} \tag{13}$$

where ω_{pi} are plasma frequencies, ω_i are resonance frequencies, f is a material parameter and ν the dissipation factor. We focus our attention on frequency regions where both $\varepsilon_i(\omega)$ and $\mu_i(\omega)$ are simultaneously negative and the corresponding index of refraction denoted by $n_i = -\sqrt{\varepsilon_i(\omega)\mu_i(\omega)}$ is also negative.

The interface response theory may be applied to explore comb-like structures, with both the waveguide and resonator having negative phase velocity. However, we have considered it appropriate to work with systems where the waveguide has a positive phase velocity and the resonator a negative one. In what follows we will use dimensionless parameters: all quantities will be normalized with respect to the plasma frequency of the combs. We have chosen the following parameters: $\omega_{p2} = 1$, $(\omega_0/\omega_{p2}) = 0.4$, $f = 0.56$. At frequencies below ω_{p2}, the dielectric response ε_2 is negative, and at frequencies $0.4 \leqslant (\omega/\omega_{p2}) \leqslant 0.603$, μ_2 is negative; therefore, in the frequency domain $0.4 \leqslant (\omega/\omega_{p2}) \leqslant 0.603$ medium 2 has a negative phase velocity. The waveguide has a frequency independent dielectric ($\varepsilon_1 = 2$) and magnetic ($\mu_1 = 1$) responses.

In figure 2 we show, for a non-dissipative system, how the number of resonators in the comb structure modifies the transmission T of electromagnetic waves. Resonances are

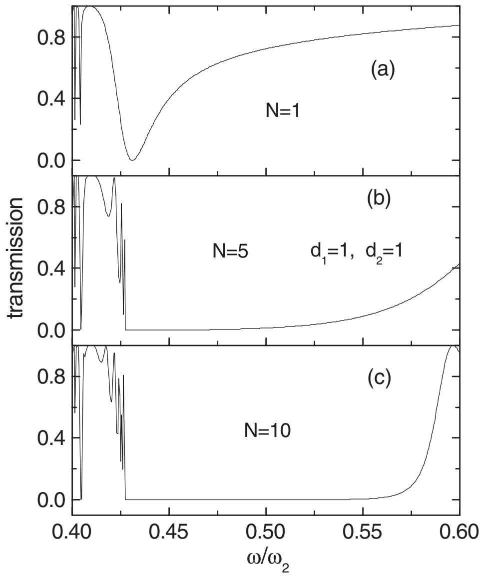

Figure 2. Transmission coefficient as a function of the dimensionless quantity ω/ω_{p2}.

manifested as dips in the transmission spectra of EM waves when the propagation wavevector satisfies equation (5). Figure 2 shows T for three different systems, where the resonator has a negative phase velocity with a finite length d_2 and the waveguide has a positive phase velocity. In this figure we take $d_1 = d_2 = 1$ in units of c/ω_{p2}. The upper panel (a) presents results for a finite waveguide with a grafted finite single resonator. The structure of the transmission is produced by the electromagnetic wave resonance according to the condition given by equation (5). In the middle panel (b) we present T for a finite comb system with $N = 5$ grafted branches. In this case, we find low frequency dips, which are also present in T of the system with only one resonator (panel (a)). In contrast, the high frequency dip of T in panel (a) evolves into a stopping gap when the number of resonators increases. Moreover, the finite value of T, for a system with a single resonator, evolves into a series of resonances when the number of resonators is increased to $N = 5$ grafted branches. For a comb-like structure with $N = 10$ grafted branches, the low frequency dips remain unchanged, and the high frequency dip evolves into a stopping gap, as displayed by the lower panel (c) of figure 2. Dips of T, which do not correspond to resonances in the resonators, are resonances of the composed systems.

To show the effects of different choices in the size of the waveguide and the resonator, we display in figure 3, for a non-dissipative system, the transmission intensity (upper panel) and the miniband structure (lower panel) of comb-like structures. For figure 3 we choose $d_1 = 1$ and $N = 5$ resonators, and $d_2 = 2$ (left panels) and $d_2 = 4$ (right panels). Dips of the transmission amplitude in panel (a1) are resonances as in panel (b) of figure 2. We also note in panel (a1) two frequency regions where the transmission amplitude vanishes (stopping gaps), which correspond to two resonances in panel (a) of figure 2. The number of resonances as well as the number of stopping minigaps increase with the resonator size.

To interpret the transmission structure we invoke panels (a2) and (b2) of figure 3, which depict the dispersion relation of the collective normal modes in the systems. In panel (a2) at low frequencies we find sharp peaks of the reduced wavevector kd_1 which correspond to dips of the transmission amplitude. At $0.425 \leqslant \frac{\omega}{\omega_{p2}} \leqslant 0.475$ there is a frequency region

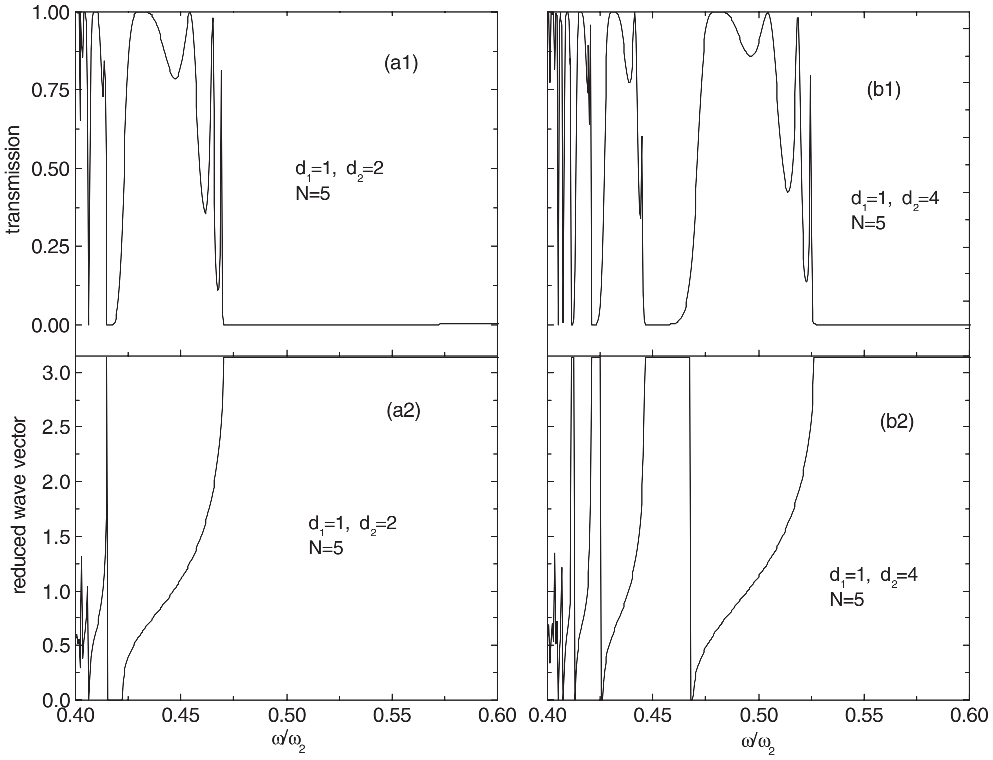

Figure 3. Transmission coefficient and miniband structure as a function of the dimensionless quantity ω/ω_{p2} and reduced wavevector kd_1. In this case we keep the size of the waveguide constant and vary the resonator size.

where the wavevector is finite; correspondingly, the transmission amplitude displays dips which correspond to resonances in the composite system. Moreover, there are two frequency regions where the wavevector becomes a complex number; these correspond to stopping gaps which yield no transmission of electromagnetic waves, in agreement with panel (a1). The increase in the resonator size yields a larger number of peaks in the dispersion relation, which in turn induces more dips in the transmission amplitude, and at the same time there is a larger number of minibands and minigaps.

We explore now, still for a non-dissipative system, the effects of keeping the resonator size constant and varying the waveguide size. Figure 4 shows the transmission spectra (upper panels) and the reduced wavevector kd_1 (lower panels) of systems with resonator size $d_2 = 2$, $N = 5$, and $d_1 = 0.5$ (left panels), $d_1 = 2$ (right panels). Similarly to figure 3, the transmission structure is interpreted in terms of the dispersion relation of the collective normal modes of the composite system. Dips of the transmission amplitude correspond to sharp peaks in the reduced wavevector kd_1. When the transmission vanishes the reduced wavevector also vanishes, yielding a structure of minibands and minigaps for the propagation of electromagnetic waves in the composite system.

Finally we illustrate the effects of dissipation in figure 5 for two values of the dissipation factor ν. As expected, the sharp peaks are smeared, but the large gap remains observable.

We have also investigated the propagation of electromagnetic waves in comb-like structures taking into account the boundary condition $E = 0$, at the extremities of the

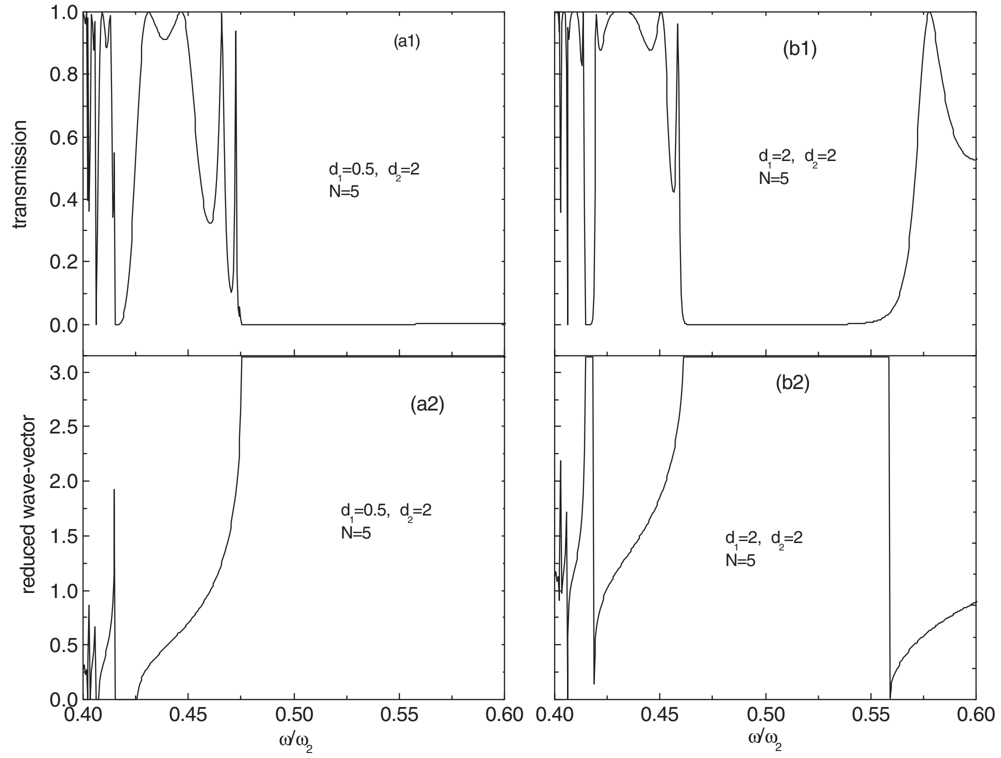

Figure 4. Transmission coefficient and miniband structure as a function of the dimensionless quantity ω/ω_{p2} and reduced wavevector kd_1. In this case we vary the size of the waveguide and keep the resonator size constant.

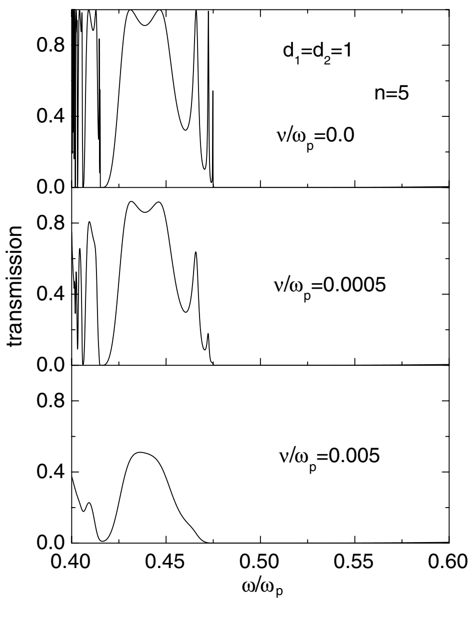

Figure 5. Transmission coefficient as a function of the dimensionless quantity ω/ω_{p2}. In this case we vary the dissipation coefficient ν.

resonators. The results (not shown) exhibit similar trends to those obtained for the same structures with the boundary condition $H = 0$.

4. Conclusions

In conclusion, we have investigated for the first time electromagnetic wave propagation in comb-like composite structures made up of dissipative negative phase velocity materials. The quasi-1D composite systems are built up of a waveguide with a periodic array of grafted segments of length d_2 distant from each other by a length d_1. We have considered the resonators to have negative phase velocity, that is, to be materials with negative index of refraction, while the waveguides were supposed to have positive phase velocity. For a system with a single resonator the transmission amplitude displays dips which correspond to resonances of the electromagnetic waves in the resonator (see figure 2(a)). As the number of resonators increases, some dips remain in the structure of the transmission T, while some of them evolve into stopping minigaps (see figures 2(b) and (c)). The structure of T for systems with more than one resonator is described in terms of the reduced wavevector of the collective normal modes, which displays peaks for the resonances of T. Zero T corresponds to non-real values of the reduced wavevector and defines stopping minigaps. The number of dips of T as well as the allowed minibands and stopping minigaps increases with the number of resonators in the composite system. Dissipation smears the sharp peaks in the transmission spectra, but the large gaps remain observable and may be of interest for future investigations and applications.

Acknowledgments

The work of GHC was supported partially by SEP-PROADU 2003-01-21-001-051, and CONACYT-BUAP Project no II010G02. GHC acknowledges the hospitality of the Université des Sciences et Technologies de Lille. The authors also acknowledge 'Le Fonds Européen de Développement Régional' (FEDER) and 'Le Conseil Régional Nord-Pas de Calais' for their support.

References

[1] Veselago V G 1967 *Usp. Fiz. Nauk* **92** 517
Veselago V G 1968 *Sov. Phys. Usp.* **10** 509 (Engl. Transl.)
[2] Pendry J B, Holden A J, Holden D J and Stewaet W J 1999 *IEEE Trans. Microw. Tech.* **47** 2075
Pitarke J M, Garcia-Vidal F J and Pendry J B 1998 *Phys. Rev.* B **57** 15261
[3] Foteinoupoulou S, Economou E N and Soukoulis C M 2003 *Phys. Rev. Lett.* **90** 107402
[4] Houck A A, Brock J B and Chuang I L 2003 *Phys. Rev. Lett.* **90** 137401
[5] Parazzoli C G, Greegor R B, Li K, Koltenbah B E C and Tanielian M 2003 *Phys. Rev. Lett.* **90** 107401
[6] Ziolkowski R W and Heyman E 2001 *Phys. Rev.* E **64** 056625
[7] Fang N and Zhang X 2003 *Appl. Phys. Lett.* **82** 161
[8] Smith D R, Schurig D and Pendry J B 2002 *Appl. Phys. Lett.* **81** 2713
[9] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[10] Nefedov I S and Tretyakov S A 2002 *Phys. Rev.* E **66** 036611
[11] Li J, Zhou L, Chan C T and Sheng P 2003 *Phys. Rev. Lett.* **90** 083901
[12] Wu L, He S and Shen L 2003 *Phys. Rev.* B **67** 235103
[13] Bria D, Djafari-Rouhani B, Akjouj A, Dobrzynski L, Vigneron J P and ElBoudouti E H 2004 *Phys. Rev.* E **69** 066613
[14] Ye Z, Zheng J, Wang Z and Liu D 2005 *Solid State Commun.* **136** 465
[15] Bah M L, Akjouj A, ElBoudouti E H, Djafari-Rouhani B and Dobrzynski L 1996 *J. Phys.: Condens. Matter* **8** 4171
[16] Vasseur J O, Deymier P A, Dobrzynski L, Djafari-Rouhani B and Akjouj A 1997 *Phys. Rev.* B **55** 10434
[17] Vasseur J O, Akjouj A, Dobrzynski L, Djafari-Rouhani B and ElBoudouti E H 2004 *Surf. Sci. Rep.* **54** 1